

FOCUS

A close-up of a human eye with a blue iris. Overlaid on the eye is a white target with concentric circles and tick marks. A blue arrow points towards the center of the target.

IMPLEMENTING THE COMMON CORE APPLYING SHIFTS TO INSTRUCTION

A classroom vignette illustrates the design principles of focus, coherence, and rigor that frame the Common Core State Standards for Mathematics.

Angeline K. Gaddy, Shannon E. Harmon, Angela T. Barlow,
Charles D. Milligan, and Rongjin Huang

I know that for the Common Core I need to rethink how I teach mathematics. But how do I get started? What should I concentrate on? What do I need to do differently?

As we have worked with teachers in professional development, these questions and others have led us to consider focus, coherence, and rigor as key areas in which teachers need to adjust their instruction. Making sense of these design principles has served as a springboard for the teachers in implementing the Common Core State Standards for Mathematics (CCSSM). Recognizing that others may be asking similar questions, we aim in this article to support teachers in not only understanding these design principles but also applying them as shifts to instruction. We begin by situating these principles within the context of NCTM's earlier Standards and CCSSM.

FOUNDATION FOR THE CCSSM

With such publications as *Curriculum and Evaluation Standards* (1989) and *Principles and Standards for School Mathematics* (2000), NCTM has played a significant role in defining a vision for school mathematics. In particular, the Curriculum Principle (NCTM 2000, pp. 14–16) described the need for students to learn important mathematics (focus) that is interconnected within and across grade levels (coherence). In addition, the Learning Principle (NCTM 2000, pp. 20–21) described the need for students not only to develop conceptual understanding and procedural fluency but also to apply this knowledge (rigor).

Building on this foundation, the creators of CCSSM used focus, coherence, and rigor as design principles to frame these standards (CCSSI 2010). That is, they sought to create a curriculum that included important interconnected mathematics and represented a balance of conceptual understanding,

procedural skill, and application. Understanding the need to support teachers in implementing CCSSM, educators and researchers have presented these design principles as *shifts*, or “key changes required by the Standards” (Student Achievement Partners 2012b, paragraph 2). Because many of us are unsure of what these standards entail in regard to our instruction (Gewertz 2013), these shifts—focus, coherence, and rigor—provide a means for understanding the instructional changes needed for implementing the Common Core.

To understand the shifts, we first offer a description of the terms *focus*, *coherence*, and *rigor*. Next, we look at how to apply these shifts in a classroom setting. Finally, we solidify this understanding through a vignette that illustrates this application of the shifts.

Focus

To meet the expectations of CCSSM, teachers must *focus* on the mathematical ideas embedded within the standards. Attention should not be limited to the development of procedural skill. Rather, conceptual understanding along with application of mathematical ideas should play a key role in students’ learning.

Coherence

CCSSM emphasizes the mathematics progression across grade levels as well as the links among mathematics topics within conceptual categories. The term *coherence* is used to describe these attributes of CCSSM. This means that “each standard is not a new event, but an extension of previous learning” (Student Achievement Partners 2012a, paragraph 2). Therefore, we must use the learning trajectories present within CCSSM to inform our teaching.

Rigor

CCSSM calls for mathematics to be taught with a level of *rigor* that includes equitable attention to conceptual understanding, procedural fluency, and application. As teachers, we must purposefully plan learning opportunities for students with this in mind. Purposeful planning includes attention to the task and its implementation. Tasks with high cognitive demand (Stein et al. 2000) hold the potential for students to achieve rigor through the development of conceptual understanding and application of mathematical ideas.

The implementation process, however, is vital. The key idea here is that when teachers focus on task selection and implementation, students will move forward in meeting rigor. A classroom example representing these ideas follows.

APPLYING SHIFTS TO INSTRUCTION

The first step toward understanding any standard is to focus on its major mathematical ideas. Therefore, we begin by identifying the conceptual understandings, procedural skills, and mathematical applications embedded within a given standard. To demonstrate this idea, we will use CCSSM Standard F-BF.2 (Functions, Building Functions), which states that students will “write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms” (CCSSI 2010, p. 70). Students will understand that sequences can be represented in multiple ways, including explicit and recursive formulas. Students will gain skill in writing both explicit and recursive formulas for given sequences. In addition, students will apply these understandings and skills to model real-world situations in which sequences occur.

Recognizing the coherence represented in CCSSM, our next step is to consider the progression of the mathematics for this standard across grade levels. According to the standards, students’ learning experiences in mathematics before high school should have addressed characteristics of a function; different representations of functions; differences between linear and nonlinear functions; and functions as a tool for modeling linear relationships. A natural extension of linear functions is arithmetic sequences (see **fig. 1**). To support coherence, instruction should begin with these ideas and then move toward the development of geometric sequences.

Our final step is to consider the level of rigor that is necessary to meet the expectations of our sample standard. First, we must select a task with high cognitive demand whose mathematical goals align with the standard. For Standard F-BF.2, we selected the Pet Ward Construction problem

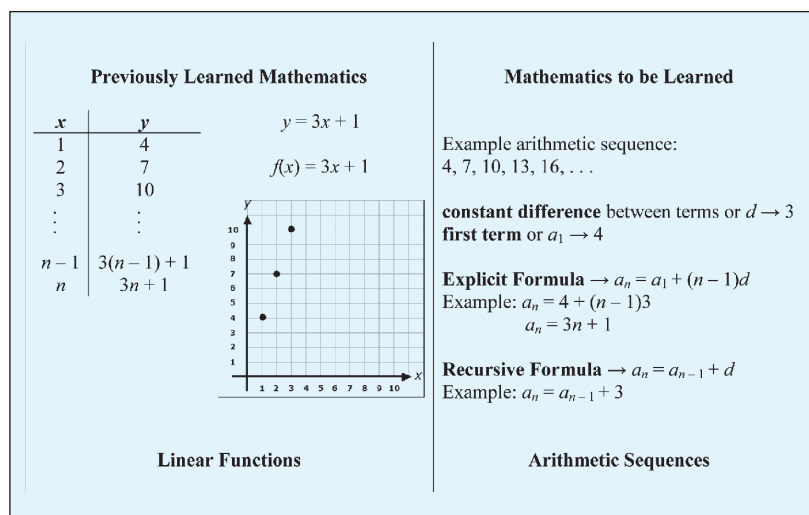
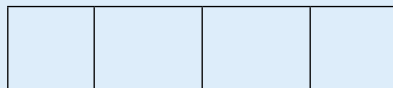


Fig. 1 The mathematics of linear functions prepares students to learn about arithmetic sequences.

A national pet-hotel chain is planning to build units for a series of franchises. Each unit for small pets is a row of two-meter-by-two-meter square wards. The wards are connected as shown in the partial floor plan below:



Walls for these units come only in two-meter panels, and the number of two-meter panels needed depends on the number of wards to be included in the unit. Because the management plans to build many units of different sizes, the manager wants to have a rule relating the number of wards and the number of panels.

Source: Heid et al. (1995), p. 50

Fig. 2 The Pet Ward Construction problem is a task with high cognitive demand.

(Heid et al. 1995) (see **fig. 2**). This task supports the conceptual understanding to be developed regarding arithmetic sequences; thus, it is necessary to consider the implementation of the task. The following classroom vignette provides a view of the shifts in instruction.

A CLASSROOM VIGNETTE

After the students read the Pet Ward Construction problem, the teacher leads a discussion. As students share their observations about the image and problem, including how many panels are used and the possibility of other arrangements of the wards, the teacher makes comments such as “Nice observation,” “Will this always be true?” and “Is there anything else we know?” After several students have contributed, the teacher asks a student to summarize what the problem is asking and then tells the class to work in groups to formulate a plan for solving the problem. As students work, the teacher distributes chart paper and markers for groups to record their ideas. She circulates around the room, attending to the strategies being used. Group 3’s representation (see **fig. 3a**) is unique. Most groups produce work similar to that featured in **figure 3b**, although not all groups develop a generalized pattern. Two groups’ work is similar to that shown in **figure 3c**.

After the groups have displayed their ideas on chart paper, the teacher brings the class together to discuss the different strategies. The strategies used by groups 1, 3, and 7 are representative of the strategies of the class as a whole, so the teacher selects the work of these three groups to present.

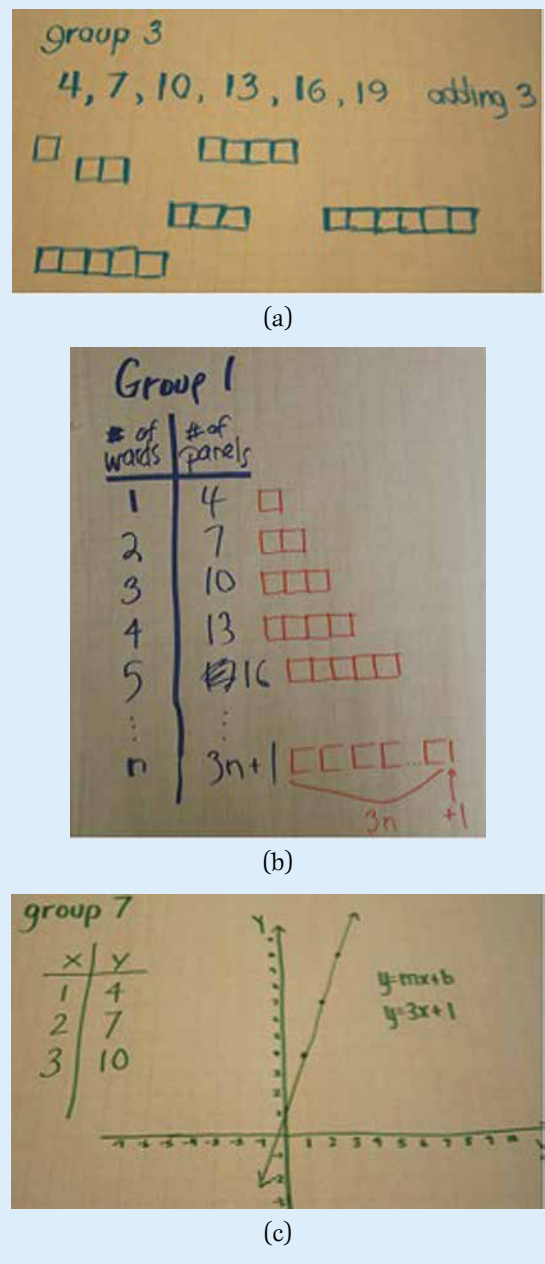


Fig. 3 Different representations show the pattern of adding 3.

Teacher: Wow! I see several different strategies that we should think about. I will ask three groups to present their work. Then I will ask you all to think about how the strategies are alike and different. Let’s start with group 3. Dave, will you present your group’s strategy for solving this problem?

Dave: We listed the number of panels for 1 ward and 2 wards and 3 wards and so on, and we noticed that we were adding 3, and we drew pictures.

Teacher: Thank you, Dave. Let’s hear from group 1. Anna?

Anna: We drew a T-chart and put the number of wards on one side and the number of panels on the other side. We also drew pictures to help us

count the number of panels. We saw what Dave was talking about that the number of panels was going up by 3, but we wanted to be able to tell the number of panels based on the number of wards.

Teacher: Would someone from Anna's group tell us about this expression, $3n + 1$? Carra?

Carra: We looked at the wards as a bunch of Cs. So, see, in the picture, the shape of a ward is a C, and each has 3 panels. So you put together all of the Cs, and then you have to add 1 more panel at the end.

Teacher: OK. The last group I would like for us to hear from is group 7. Juan, will you present your group's work?

Juan: We also drew a T-chart but then made a graph so that we could find the slope and y -intercept. So we wrote an equation.

After the different strategies have been shared, the teacher leads a discussion of these. A sampling of questions that the teacher used to guide the discussion follows:

THIS PROBLEM-BASED APPROACH TO INTRODUCING THE CONCEPT OF ARITHMETIC SEQUENCES ENHANCED STUDENTS' ABILITY TO APPLY THE KNOWLEDGE TO REAL-WORLD PROBLEMS.

1. How are all three solution processes the same?
2. How are the solution processes different?
3. Why is it that we see the phrase "adding 3" on group 3's poster, but on the other two posters it looks like we are multiplying by 3?
4. If we needed to know how many panels to purchase for 100 wards, how would you figure that out using group 7's method? Using group 1's method? Using group 3's method?

As the end of class nears, the teacher asks the students to write their thoughts regarding the different solution methods. In thinking about the next day's lesson, she plans to build on the students' presentations by introducing the terms *arithmetic sequences* (as depicted in group 3's poster), *recursive formulas* (building on group 3's phrase of "adding 3"), and *explicit formulas* (building on group 1's generalized pattern and group 7's equation).

DISCUSSION OF VIGNETTE

This vignette demonstrates how the teacher purposefully pursued rigor by appropriately implementing a task and orchestrating the discussion

of students' work (Stein et al. 2008). Initially, the teacher used questions to support students in making sense of the problem. Then she moved around the room, monitoring students' solution processes with a focus on selecting and sequencing different strategies for presentation. Finally, she asked students to explain and justify their solutions.

In sequencing the different strategies for presentation, the teacher chose to move from the least sophisticated strategy to the most sophisticated strategy. Group 3's strategy (see **fig. 3a**) was an "informal" but natural way of presenting the "adding 3" structure based on the students' previous experiences in pattern seeking. Featuring this strategy for the initial presentation not only validated the strategy but also likely supported any struggling students in developing a fundamental understanding of the mathematics represented in the problem. Next, the teacher chose to present group 1's work because its strategy represented the thinking of the majority of the class. Group 1's strategy (see **fig. 3b**) focused on expressing the "adding 3" pattern using an algebraic

expression with the support of a geometric representation. Finally, the teacher presented group 7's strategy because it represented the most sophisticated strategy. This strategy involved using a linear equation and graph to represent the "adding 3" structure (see **fig. 3c**). Although this particular teacher chose to move from least to most sophisticated strategy, this sequence is not always necessary. For a discussion of purposeful sequencing of students' presentations, see Stein and Smith (2011).

Following the presentations, the discussions of these strategies supported students' in recognizing the repeated structure using different representations or tools. Moreover, questions 1 and 2 helped students understand that there are multiple ways to represent the same pattern (repeated structure) from different perspectives—focusing on the relationship between the current term and the previous term or focusing on the relationship between the current term and the term number. Question 3 led students to translate between the recursive formula and the explicit formula and determine the advantages and disadvantages of each. Finally, question 4 encouraged students to examine the power of using the explicit formula.

These discussions focused on the development of students' conceptual understanding of recursive and explicit formulas, their usefulness in modeling situations, and the translation between the two forms. Through this process, multiple ways of representing the same pattern of "adding 3"—tables, algebraic expressions, geometric representations, and graphs—supported students in developing procedural fluency with algebra and functions. This problem-based approach to introducing the concept of arithmetic sequences enhanced students' ability to apply the knowledge to real-world problems. The teacher demonstrated her intent to focus on the core concept of arithmetic sequences coherently. Exploring the task in this lesson built on students' previous knowledge of pattern seeking, algebraic expressions, and linear equations and graphs and developed students' understanding of recursive and explicit formulas for arithmetic sequences. In this way, the lesson demonstrates coherence. This exploration laid the foundation for learning arithmetic sequences formally and extensively in the next lessons. In this way, the three Common Core shifts—focus, coherence, and rigor—are demonstrated.

SUPPORTING MATHEMATICS INSTRUCTION

For teachers to better understand instructional changes needed to meet CCSSM expectations, viewing instruction through the lenses of focus, coherence, and rigor is beneficial. By representing these shifts through descriptions and application, we aimed to support teachers in implementing CCSSM. Although these standards served as the impetus to the discussion, these shifts represent ideas that will support quality high school mathematics instruction regardless of the curriculum.

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ANGELINE K. GADDY, akg3c@mtmail.mtsu.edu, is a doctoral student studying mathematics education at Middle Tennessee State University in Murfreesboro.



Formerly a secondary school mathematics teacher, she is interested in supporting preservice and in-service mathematics teachers in improving their teaching skills.



SHANNON E. HARMON, shannon.harmon@mtsu.edu, works with both preservice and in-service teachers at MTSU. She is interested in providing teachers and teacher candidates with a vision of re-



formed-based teaching. **ANGELA T. BARLOW**, angela.barlow@mtsu.edu, is the director of the Mathematics and Science Education Ph.D. program at MTSU. She is interested in developing teachers' beliefs that align with reform-oriented instruction.



CHARLES D. MILLIGAN, charles.milligan@mtsu.edu, is an assistant professor in the Educational Leadership Department at MTSU. He is a former principal and superintendent in K-12 education, and his research interests include evidence-based accountability. **RONGJIN HUANG**, rhuang@mtsu.edu, is an associate professor of mathematics education at MTSU. His research interests include mathematics classroom research, mathematics teacher education, and comparative mathematics education.